Cosmology: The Homogeneous Universe

1. The principle of containment and of location

*Cosmology* describes the evolution of the Universe.

Cosmology in the past was concerned with the **center** and **edge** of the Universe.

**The principle of containment:**

*The physical universe contains everything that is physical and nothing else.*

**Literature:** Harrison, Edward
Cosmology, The Science of the Universe
• We, as physical creatures are part of the physical universe.

• **Spacetime**, the 4-dimensional continuum of space and time is a physical reality

Space and time are a **property** of the universe and do not extend beyond.

The universe contains space and time. It does not exist in space and time.

• The universe has **no edge** (curved or infinite).

• **Time** did not exist „before“ the creation of the Universe, i.e. like in a black hole, time has an edge.
The location principle:

*It is unlikely that we have a special location in the physical universe.*

- The Earth, the Solar System and the Galaxy are certainly unique.
- However, other civilisations on other Earths have also unique locations.
- Uniqueness of location does not mean that the place is special when all local details are ignored.
- The cosmic center would be such a special location.
- Suppose we observe an isotropic Universe:

![Diagram showing unique isotropic place, unlikely, and more likely to live on a sphere]
2. Evidence for an isotropic universe

On small scales the universe appears very complex.
Slices through the local Universe
• On scales of a few 100 million pc the universe becomes \textbf{isotropic}

• The APM-Survey: 2 million galaxies in the southern sky
**Isotropy and homogeneity:**

- A **moving** observer would be able to test the **homogeneity** of the universe.
- A **fixed** observer can just determine its **isotropy**.

\[\text{isotropic} \quad \rightarrow \quad \text{homogeneous}\]

- An **inhomogeneous** universe can still be **isotropic** at certain points.
- An **anisotropic** universe can be **homogeneous**.

**Note:** a homogeneous universe is isotropic everywhere if it is isotropic at one point.

**But:**

- Local isotropy + special location is highly **unlikely** in an anisotropic universe!

**The universe is probably homogeneous**
3. The expanding universe

- Relative to \( O \), \( P \) moves with \( \mathbf{v}(r) \)
- Relative to \( O' \), \( P \) moves with \( \mathbf{v}'(r') \)
- Relative to \( O \), \( O' \) moves with \( \mathbf{v}(a) \)

\[
\mathbf{r}' = \mathbf{r} - \mathbf{a} \\
\mathbf{v}'(r') = \mathbf{v}(r) - \mathbf{v}(a)
\]

Homogeneity:

\[
\mathbf{v}(r') = \mathbf{v}'(r') \quad \Rightarrow \quad \mathbf{v}(r - a) = \mathbf{v}'(r') = \mathbf{v}(r) - \mathbf{v}(a)
\]

Solution:

\[
v_i(r, t) = \sum_{k=1}^{3} a_{i,k}(t) x_k
\]

Isotropy:

\[
a_{i,k} = 0 \quad \text{for} \quad i \neq k \quad \text{and} \quad a_{ii} = H(t)
\]

\[
v_x = H \cdot x; \quad v_y = H \cdot y; \quad v_z = H \cdot z
\]
\[ \vec{v}(r) = H_0 \, r \]
Solution:

Each observer finds, that all other objects move away (H > 0) or approach (H < 0) with a velocity that is proportional to their distance.

**The scale factor R of the Universe:**

\[ \vec{r}(t) = \vec{r}(t_0) \times \frac{R(t)}{R_0} \]

**Hubble Constant:**

\[ \vec{v} = \dot{\vec{r}} = \frac{\vec{r}(t_0)}{R_0} \cdot \dot{R} = H \dot{r} = H \frac{\vec{r}(t_0)}{R_0} \cdot R \]

\[ H = \frac{\dot{R}}{R} \]
The Cosmological Redshift

• Hubble showed 1929, that the spectral lines of galaxies are redshifted.

Redshift:
\[ z = \frac{\lambda_0 - \lambda}{\lambda} \]

• He interpreted this redshift as a Doppler shift.

Radial velocity:
\[ \frac{v}{c} = \frac{\Delta \lambda}{\lambda} \quad \rightarrow \quad v = c \times z \]
The Hubble Constant:

\[ \vec{v}(r, t) = H(t) \vec{r} \]

Hubble (1929)

The Hubble constant today is usually written as

\[ H_0 = H(t_0) = h \times 100 \text{ km/s/Mpc} \]

Note, that \( H \) is not a constant in time!

\[ H_0 = 71 \text{ km/s/Mpc} \]
\[ r = \frac{v}{H_0} = \frac{c \times z}{H_0} \approx 5000 \cdot z \text{ Mpc} \]

\[ H_0 = 71 \text{ km/s/Mpc} \]

- For \( z = 1 \), that is \( r > 5000 \text{ Mpc} \) galaxies would move faster than the speed of light.

- This is not allowed \( v \) becomes constant for large distances.

This is in contradiction with the homogeneity of the universe
GR effects should lead to distortions close to $z=1$ which are not observed.

- **Space** between the galaxies **expands**.

- Galaxies are **stationary**.

**Redshift is not a Doppler shift**

- When light travels through **expanding space**, its wavelength is stretched:

  \[
  \lambda_0 = \lambda_{\text{emit}} \times \frac{R_0}{R_{\text{emit}}}
  \]

  \[
  z = \frac{\lambda_0 - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{R_0}{R} - 1
  \]

- The **redshift** $z$ is independent of velocity.
- $z$ measures directly the **amount of expansion** of space since emission.
- $z$ is independent of the **time dependence** of the expansion.
4. World models \( R(t) \), curved space and the size of the Universe

The Einstein equations of general relativity:

\[
R_{ij} - \frac{1}{2} g_{ij} R = 8\pi G T_{ij} + \Lambda g_{ij}
\]

- **Ricci tensor**: space time curvature
- **metric tensor**
- **energy-momentum tensor**: mass and energy
- **cosmological constant**
- **Ricci scalar**: \( R = g^{ik}R_{ik} \)
The field equations lead to the two **Friedmann equations**:

\[
\dot{R} = -\frac{4\pi G}{3} \left( \rho + 3 \frac{p}{c^2} \right) + \frac{1}{3} \Lambda R
\]

\[
\dot{R}^2 = \frac{8\pi G}{3} \rho R^2 + \frac{1}{3} \Lambda R^2 - k \frac{c^2}{R_0^2}
\]
4. World models $R(t)$, curved space and the size of the Universe

Curvature:

$$K = \frac{k}{R^2}$$

with

- $k=1$, elliptical space
- $k=0$, flat space
- $k=-1$, hyperbolic space

$K=0$: flat plane
$K>0$: elliptical plane
$K<0$: hyperboloid
\[ \dot{R}^2 = \frac{8\pi G \rho}{3} R^2 - k \frac{c^2}{R^2} = \frac{2GM}{R} - k \frac{c^2}{R^2} \]

Total energy: \[ E = \frac{1}{2} \dot{R}^2 - \frac{GM}{R} = -k \frac{c^2}{2R_0^2} \]

Elliptical universe \((k > 0)\): \( E < 0 \)
Parabolic universe \((k = 0)\): \( E = 0 \)
Hyperbolic universe \((k < 0)\): \( E > 0 \)
Dark energy and globular clusters:
The Big Bang

Planck time: $10^{-43}$ s after the Big Bang

Temperature: $10^{35}$ K

Density: $10^{94}$ g/cm$^3$

The World of Quantum Gravity
Can we see the big bang?

The fluctuations in the CMB are determined by the basic parameters and matter content of the Universe.
The cosmic microwave background: echo of the big bang

• 1965 Arno Penzias and Robert Wilson (Bell Labs) detected the low-temperature cosmic microwave background radiation

\[ T = 2.728 \, \text{K}, \quad \lambda = 1 \, \text{mm}, \, \sim 400 \, \text{photons/cm}^3 \]

WMAP mission:
Cobe image of the CMB

\[ \frac{\delta T}{T} = 0.002 \]

\[ v = 600 \text{ km/s} \]

\[ \frac{\delta T}{T} = 0.0001 \]

\[ \frac{\delta T}{T} = 0.00001 \]
After substraction of the Earth’s motion and contribution of the **Galaxy** the CMB is remarkably isotropic.

It reflects **baryonic density inhomogeneities** about 100 000 yrs after the big bang at a redshift of $z=1000$

Perfect black body radiation

\[
T = 2.728\, \text{K} \quad \quad \frac{\delta T}{T} \approx 0.00001
\]
Planck CMB: is the Universe *really* homogeneous?
Evolution of density perturbation

Gravity decelerates the expansion

$z = 1000$
Evolution of observed density perturbations

\[ \frac{\delta \rho}{\rho} = 0.00001 \quad \text{at } z = 1000 \]

\[ \frac{\delta \rho}{\rho} = 10^{-2} \quad \text{at } z = 0 \]
Evolution of density perturbations with dark matter

380000 yrs
Evolution of density perturbations with dark matter

380000 yrs

today
The origin of structure

221283 galaxies

15 May 2002
The „Cold Dark Matter“ - Model

The favored candidates for dark matter are hypothetical elementary particles that are *long-lived, cold* and *collisionless*.

**Collisionless:**
- no internal structure
- only gravitationally interacting

**Cold:**
- non-relativistic velocities
In contrast to models of hierarchical clustering, **dark matter cores** appear to be **isothermal with a flat density distribution**.
The Cosmological Substructure Problem

(Moore, 2001)
The Cold Dark Matter's Satellite Problem

Cluster of galaxies

Single galaxy

predicted for clusters and galaxies

observed in galaxies

Moore et al. 1999
How can we solve the dark matter core problem?

1. **CDM does not exist** (MOND).

2. **Violent decoupling of gas and dark matter**:
   - Supernova-driven galactic winds expell most of the gas.
   - This event reduces also the CDM core densities.
   - Most of the dark matter substructures would be dark.

3. **Triaxial dark matter halos** (Hayashi et al. astro-ph/0408132)

4. **Revised models for cold dark matter** (Ostriker & Steinhardt 04)
   - strongly self-interacting
   - self annihilating
   - repulsive
   - warm
   - fuzzy
   - decaying
MOND

\[ g = \frac{GM}{r^2} \cdot f\left(\frac{g}{g_0}\right) \]
Modification of Newton’s gravity law at an acceleration:
MOND

Modified Newtonian Dynamics
Modification of Newton’s gravity law at an acceleration: